CHAPTER 9

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MATHEMATICS TEACHERS' BELIEFS AND EXPERIENCES WITH INNOVATIVE CURRICULUM MATERIALS

The Role of Curriculum in Teacher Development

Abstract. This chapter draws attention to the educative potential of teachers' experiences with the curriculum materials that have been developed in the context of recent efforts to improve K-12 mathematics education in the United States. Examples from two different professional development settings illustrate how teachers' beliefs can change on the basis of experiences with these innovative curriculum materials. Discussion of these examples suggests the need for greater attention to teachers' beliefs about mathematics curriculum.

1. INTRODUCTION

In the United States and many other parts of the world, ongoing curriculum initiatives aim to revise the conventional view of mathematics learning as the mastery of a fixed set of facts and procedures to more centrally locate the processes of investigation, sense-making, and communication in classroom activities. At the heart of such reforms is the distinction between *inquiry mathematics* and *school mathematics* (Cobb, Wood, Yackel, & McNeal, 1992). In contrast with traditional classroom activities that emphasize repetition, practice, and routinized means to some focused endpoint, inquiry mathematics instruction emphasizes student engagement in problem-solving and theory-building about important mathematical situations and concepts.

Bringing about such dramatic changes in mathematics instruction demands that teachers possess beliefs about mathematics, learning, and teaching that depart significantly from school mathematics traditions (Battista, 1994; Thompson, 1992). Perhaps the greatest obstacle for teachers is a lack of personal familiarity with mathematical problem-solving and sense-making – processes that most have never experienced themselves, as students or teachers. Even when teachers' efforts to emphasize inquiry mathematics are supported by specially designed curriculum materials, teachers often struggle to bring about significant changes in classroom practice (Cohen, 1990; Grant, Peterson, & Shojgreen-Downer, 1996; Wilson &

G. C. Leder, E. Pehkonen, & G. Törner (Eds.), Beliefs: A Hidden Variable in Mathematics Education? 149-159.

Lloyd, 2000). Because many teachers' beliefs and practices are deeply tied to school mathematics traditions, the success of current mathematics education initiatives depends on our identification of viable ways to encourage and enable teachers to make significant shifts in their beliefs.

The purpose of this chapter is to draw attention to the educative potential of teachers' experiences with the curriculum materials that have been developed in the context of recent efforts to improve K-12 mathematics education in the United States. Of particular interest is how teachers' beliefs can *change* on the basis of experiences with these innovative curriculum materials. The chapter begins by outlining how teachers' beliefs relate to the visions of instruction advocated in recent years. A subsequent section distinguishes the new curriculum materials from traditional texts. Next, two sections offer discussions and illustrations of two viable contexts for using innovative curriculum materials to help teachers develop reform-oriented beliefs about mathematics pedagogy. A final section calls for greater attention to teachers' beliefs about mathematics curriculum.

2. TEACHERS' BELIEFS AND INQUIRY MATHEMATICS INSTRUCTION

The responsibilities of mathematics teachers are extensive. In one of their influential reform documents, the National Council of Teachers of Mathematics [NCTM] (1991) delineates four categories of mathematics teachers' work: setting goals and selecting or creating mathematical tasks to help students achieve these goals; stimulating and managing mathematical discourse; creating a classroom environment to support teaching and learning mathematics; and analyzing student learning, the mathematical tasks, and the environment in order to make ongoing instructional actions. Teachers' beliefs about and knowledge of mathematical subject matter are critical influences on how they cope with the challenges of classroom instruction (Brophy, 1991; Chapman, this volume; Fennema & Franke, 1992; Thompson, 1992). The review by Wilson and Cooney (this volume) suggests that there is a complex relationship between beliefs and classroom practices. Whereas many teachers learned mathematics by memorizing rules, they must now learn to view rich mathematical understanding as the capacity to use mathematics to reason, to communicate, and to pose and solve meaningful problems (Hiebert et al., 1996; NCTM, 1989, 1991, 2000). Doing so involves learning that a mathematical idea or solution should be judged to be appropriate or correct because it is meaningful and works, not just because the teacher, textbook, or some other outside authority says it is so (Cooney, 1994; Wilson & Goldenberg, 1998; Wilson & Lloyd, 2000).

Mathematics is learned through an active, social process of construction (Cobb, 1995; Davis & Maher, 1990; Mathematical Sciences Education Board [MSEB] and National Research Council [NRC], 1989; von Glasersfeld, 1984). Helping teachers to make sense of constructivist learning theories is a major challenge for those involved with the professional development of teachers. A teacher's beliefs about how students engage in mathematical activity and learn are critical factors in the ability and tendency to design and carry out inquiry-based instruction. Researchers

on the Cognitively Guided Instruction (CGI) project have promoted the theory that teacher development involves a fundamental change in the content and organization of teachers' knowledge about children's mathematical thought (Fennema et al., 1996). Teachers need to have a sense of how student understanding develops so that they can anticipate what sorts of mathematical activities will help specific students' learning (Even & Tirosh, 1995). For most teachers, development of this sense will involve a shift in how they conceptualize the mathematical learning process. Understanding learners and subject matter as interactive is one of the most important beliefs about teaching. Ball (1993) describes this conception as a "bifocal perspective – perceiving the mathematics through the mind of the learner while perceiving the mind of the learner through the mathematics" (p. 159). For teachers to appreciate and strive for this relationship is a major challenge of reform-oriented teacher development programs.

Inquiry about the nature of teachers' learning from experiences with innovative curriculum materials relies upon analysis of teachers' mathematical and pedagogical beliefs. Entwined within these beliefs are teachers' beliefs about mathematics curriculum. A better understanding of teachers' beliefs about mathematics curriculum is vital to the success of current reform efforts. Although textbooks have long held prominent roles in guiding practice in American classrooms (Tyson-Bernstein & Woodward, 1991), we know surprisingly little about how teachers' beliefs about curriculum materials relate to their beliefs about mathematics, teaching, and learning, and how they develop during teacher education and school-based experiences.

3. REFORM-ORIENTED MATHEMATICS CURRICULUM MATERIALS

To support teachers in reforming mathematics classroom activity, numerous sets of reform-oriented curriculum materials have been developed in the United States (e.g., Investigations of Number, Data, and Space; Connected Mathematics Project; Core-Plus Mathematics Project; Interactive Mathematics Program; Mathematics in Context; etc.). Although the curriculum materials of these programs incorporate specific aspects of reform recommendations in diverse ways (emphasizing different themes or activities), the materials share certain qualities that distinguish them from traditional mathematics textbooks.

First, reform curricula explicitly incorporate reform ideas about mathematics and pedagogy by emphasizing inquiry mathematics: student explorations of real-world mathematical situations and discussions of problem-centered activities. Furthermore, the materials are formatted to support these mathematical and pedagogical differences. American texts are typically divided into chapters outlining self-contained daily lessons for the teacher to present (composed primarily of definitions and examples of the lesson's content) followed by practice exercises for the student. In contrast, most reform-oriented curriculum materials are published in unit booklets (offering greater flexibility in ordering) that pose large-scale problems and situations, centered on particular mathematical themes and content areas, for students to investigate and debate.

A second substantive difference is that reform-oriented materials generally offer more extensive information for teachers than do traditional texts. In addition to providing problem solutions, the teachers' guides for most of these new materials offer details about different representations of content, historical information about mathematical and pedagogical ideas, examples of what students might believe or understand about particular activities and content, potentially fruitful questions for eliciting discussion, and so on. The inclusion of these details has been motivated in part by the failure of the "teacher-proof" curriculum materials of the 1950s and 1960s to facilitate substantial educational change. After all, it is *teachers* who determine how the innovations envisioned by reformers and curriculum designers become implemented in mathematics classrooms (Cooney, 1988; Freudenthal, 1983).

3.1. Contexts for Teacher Learning with Reform-Oriented Curriculum Materials

There exist many potential ways that teachers can learn from engagement with innovative K-12 mathematics curricula. During inservice workshops or preservice courses at the university, teachers may work collaboratively "as students" on the mathematical lessons outlined in the materials. Doing so can offer teachers critical opportunities as learners because they can personally experience unfamiliar mathematics in novel ways. Another rich context for educative experiences with curricula is teachers' own classrooms. As teachers implement curriculum materials in their classrooms (or as student teachers), they may develop new mathematical and pedagogical beliefs and skills on the basis of their design of lessons, interactions with students, use of technology, and so on. As Llinares (this volume) explains, these contexts (university and school practice) offer quite different situations for learning. The next two sections illustrate how these two contexts offer unique and useful ways for teachers to learn with curriculum materials.

4. TEACHER LEARNING WITH CURRICULUM MATERIALS IN THE MATHEMATICS CLASSROOM

When implementing innovative curriculum materials in K-12 classrooms, teachers are afforded frequent and extensive learning opportunities. Because the act of teaching (regardless of the context - reform or otherwise) is a learning process, instruction necessarily impacts teachers' beliefs. As Ball (1994) describes, teachers continually construct new knowledge from classroom experiences:

> Teachers must figure things out as they teach. They are constantly faced with the data of their own experience. They must develop knowledge of particular children, of the material they are teaching, and of ways to engage students in the content. (p. 9)

The potential for learning from classroom experiences increases as teachers attempt to enact reform visions. Existing beliefs and practices may be directly challenged by the process of interpreting and implementing reform recommendations and curricula. In other words, teachers' sense of efficacy may be brought into question when confronted with novel curricular materials (Philipou & Christou, this volume).

Implementation of innovative materials offers one potentially powerful site for teachers to learn about themselves, their students, mathematics, and the teaching and learning of mathematics (Ball & Cohen, 1996; Lloyd, 1996; Russell et al., 1995).

Consider, for example, the experiences of Mr. Allen, a high school mathematics teacher in the United States who was studied for several years as he taught about functions with reform-oriented curriculum materials (Lloyd, 1996; Lloyd & Wilson, 1998; Wilson & Lloyd, 2000). The Core-Plus curriculum unit that he implemented required students to create and analyze multiple representations (tables, graphs, sentence descriptions, equations) for a variety of real-world situations. In the first year that he used the new materials, Mr. Allen demonstrated a strong preference for graphical representations of functions. His discussions with students reflected this tendency. For example, as he said to his class on one occasion, "The table gives you times and heights, but the graph gives you *the relationship* between time and height". After teaching with this curriculum unit, Mr. Allen's beliefs that graphs offer the optimal display of a relationship *changed.* At the beginning of his second year, he articulated much richer views of different representations. As he explained,

> With a graph it's visual and you can see the pattern, but you don't necessarily have right in front of you the actual pairings of the data to be able to look critically at "This is an x with a y" and "This is a new x with a y." ... If there is some constant increase, with a graph you can see that it's a line, but you don't necessarily see the data that might be able to tell you exactly a specific rate of change. Having the equation allows you to generate values of the function quickly and maybe interpolate what might be happening down the road for some piece of data x where you might not see all of that in just a small portion of the graph or just in five or six pieces of data.

These comments are consistent with his classroom instruction in year 2, when he treated the different representations more equitably in his conversations with students. For instance, as he explained to his class, "There are many ways to take a look at a function: there's a graph, there's a table, there's a word expression, or you can get an equation or a rule".

What was the reason for this change in Mr. Allen's beliefs? There are several: While interacting with students, he came to appreciate that they did not all share his belief that graphs provide the optimal display of a relationship. He had also learned himself about mathematical situations in which a non-graphical representation is more useful. Moreover, he had become familiar with this curricular approach which emphasizes multiple representations of functional relationships. From Mr. Allen's perspective, his change was directly related to his experiences teaching with the new curriculum materials. As he explained during an interview:

> Traditional textbooks tended to emphasize the equation and the graph, but not so much the table. You get away from doing the table and you try to learn quick ways to take an equation to a graph. I mean they always tell you, "Don't make a table. This is the better way, there's a slope and a y-intercept." . . . Using the graphic calculator allows you to take a look at the change in the variables as you look at the table. It's right there in front of you and you don't have to go through the messy computation of creating the table. You're focusing on trying to take a look at how x is changing and how y is changing and if there's some pattern there.

As these examples suggest, curriculum implementation offered a context in which Mr. Allen was challenged to change his beliefs as he learned to teach with the new materials.

Mr. Allen's case illustrates how engagement with innovative curriculum materials allows teachers to learn or re-learn mathematical subject matter currently recommended for school mathematics. Whether it occurs in the classroom while teaching with novel curriculum materials or in university settings while working on curricular activities as students (as discussed below), for most teachers, this learning involves revisiting mathematical ideas that they learned as students to extend their knowledge to include more conceptual or relational understandings. Teachers' learning may also involve exploring entirely new mathematics. For example, many teachers have never learned about probability, statistics, and discrete mathematics – areas now viewed as "big ideas" in the K-12 curriculum. Because reform-oriented curriculum materials have been designed to include these topics, and emphasize conceptual connections, they are an excellent source of mathematical activities that can give these teachers first-hand experiences with the types of mathematics they are expected to teach. Furthermore, reform-oriented curriculum materials portray mathematics as a vibrant and useful subject to be explored and understood. When teachers learn personally from their work with these materials, or share in their students' engagement with these materials, they are better prepared to make, and more personally invested in making, important changes in their views of appropriate mathematical activity for the classroom.

5. TEACHER LEARNING WITH CURRICULUM MATERIALS IN UNIVERSITY AND INSERVICE SETTINGS

A useful preservice and inservice activity involves inviting teachers to work carefully through the mathematical activities presented in innovative K-12 curriculum materials. Engagement with curriculum *as learners* invites teachers to think about challenging mathematics and the nature of mathematical activity, reflect on the process of learning mathematics to develop empathy for future students, and contemplate teaching mathematics to create new personal visions of classroom practice. Such experiences are critical:

> Teachers themselves need experiences in doing mathematics – in exploring, guessing, testing, estimating, arguing, and proving ... they should learn mathematics in a manner that encourages active engagement with mathematical ideas. (MSEB & NRC, 1989, p. 65)

As teachers revisit mathematical content from new perspectives, they can begin to translate the knowledge developed as learners into pedagogical content knowledge – knowledge of mathematics *for teaching* (Shulman, 1987). Teachers can also begin to revise their views of the types of activities that give rise to rich mathematical understanding, and their views of what constitutes evidence of student understanding. Davenport and Sassi (1995) report that the veteran teachers in their study were profoundly affected by reading detailed classroom narratives in curriculum materials and other print resources. Such detailed descriptions of lessons,

including important images of students engaging in meaningful mathematics, helped the teachers to develop visions for their own classrooms. It is likely that preservice and beginning teachers would also benefit in multiple ways by reading and reflecting on the variety of information provided in innovative curriculum materials.

Consider an example from a university mathematics class for preservice elementary teachers in which middle school materials were used (Lloyd & Frykholm, 2000). In this class, without exception, every student (preservice teachers) came to recognize that there was mathematical content in the curriculum materials (intended for grades 5-8) with which they were unfamiliar or uncomfortable. Some students indicated that they preferred middle school mathematics the first time (when they were in middle school) and wished they were not asked to relearn it. However, most students appreciated the novelty of the activities in which they engaged. For instance, Stella, one of the preservice teachers, expressed,

> I am learning about how to look for reasons and explanations as opposed to simply believing "the rules" that some really ancient dead guy came up with. I prefer being able to use my own mind in solving problems. This class seems to use more common sense instead of book smarts.

This comment suggests that Stella was experiencing a different kind of mathematics than she had known before, and was developing new beliefs about what counts as "doing mathematics".

Experiences of learning and teaching with innovative curriculum materials may compel teachers to recognize that the nature of mathematics communicated in the classroom is intimately linked to the way it is shared with students. If teachers wish to communicate vibrant and useful images of mathematics, they must incorporate a range of pedagogical strategies that engage students in genuine problem-solving and problem-posing. Preservice teachers, whose process of learning to teach in reformed ways is compounded by the pressures of teaching for the first time, may greatly benefit from explicit attention to the development of models of practice during teacher education experiences. Reform-oriented curriculum materials offer useful images of what reformed mathematics teaching can look like (Davenport & Sassi, 1995).

As a field we know very little about how teachers can learn to center their instructional plans on student development. How do teachers come to view student learning as both the goal and guide of their mathematics classroom practices? A worthy area for research is the potential for teachers' own experiences doing mathematics in reform-oriented ways to support their development of instructional practices that honor and build on students' understandings. Teachers' reflection on experiences learning mathematics with reform-oriented curriculum materials can allow teachers to extract important theories about the nature of the mathematical learning process. Doing so may help them to recognize the significance of the learning that can occur during inquiry and student-centered activities. Further, when they work with teachers' editions (which include descriptions of possible student responses or work) and use innovative curricula to plan instructional activities,

teachers may learn about the processes through which students develop understandings during particular classroom activities.

6. BELIEFS ABOUT MATHEMATICS CURRICULUM

Curriculum-based professional development has the potential to involve and impact teachers' beliefs about mathematics, student learning, and mathematics pedagogy, as well as their beliefs about mathematics curriculum. This branch of teachers' beliefs is intimately related to beliefs about mathematics, learning, and teaching. Teacher learning *about* curriculum is one goal of learning *with* innovative curriculum materials. For instance, reflection on the design and uses of curriculum may help teachers to develop rich beliefs about the role of curriculum in mathematics teaching.

Let us consider again the cases of Mr. Allen and Stella. After teaching with new curriculum materials, Mr Allen was able to look at the traditional curriculum from a distance. He described the traditional curriculum as "just the way we've always done things". Experiencing something different enabled Mr. Allen to see qualities of the "old way" that had been invisible to him before. What did Stella learn about mathematics curriculum? Recall her suggestion that, when working with reform-oriented curriculum materials, she uses "[her] own mind in solving problems . . . more common sense instead of book smarts". It is noteworthy that Stella refers to "book smarts": in contrast to other experiences where the textbook is a primary source of mathematical authority, her work with these materials allowed her to recognize herself as a sense-making authority. Understanding mathematics demands much more than reproducing rules found in textbooks, and as Stella indicates, curriculum materials can provide the basis for conceptual exploration and knowledge.

The notion that teachers possess beliefs about curriculum is certainly not new. For instance, Shulman (1987) identifies "curriculum knowledge, with particular grasp of the materials and programs that serve as 'tools of the trade' for teachers" (p. 8). However, in the present climate of reform in mathematics education, teachers' beliefs about curriculum have seldom been discussed. Because textbooks and curriculum materials are often teachers' sole contact with reform visions (Ball & Cohen, 1996), this neglect is particularly alarming. Given the prominence of textbooks in teachers' classroom decision-making (Bush, 1986; Tyson-Bernstein & Woodward, 1991), we would be wise to attend more fully to teachers' beliefs about mathematics curriculum and their role in the teacher change process. Reform recommendations and associated curriculum materials cannot and do not bring about change alone – educational change is a complex human endeavor involving teachers *and* texts (Cooney, 1988; Freudenthal, 1983).

Teachers' beliefs about mathematics curriculum include more than a familiarity with the currently available materials for designing mathematics instruction. Beliefs about curriculum encompass understandings of the role of curricular materials in the teaching and learning process, the philosophies of teaching and learning that underlie diverse curriculum materials, knowledge of the appropriateness of

particular materials for certain classes and individuals, and the practical and intellectual understandings necessary for making adjustments to curricular approaches. The notion that a textbook outlines *one* of many possible mathematical and pedagogical approaches is central to teachers viewing curriculum as adaptable. Teachers are often dissatisfied with features of textbooks and curriculum materials but tend not to change or adapt those features (e.g., Lloyd, 1999). Teachers' treatment of curriculum as fixed suggests that teachers may struggle to conceive of curriculum as a flexible guide that permits and encourages alterations with respect to the changing needs and demands of particular students. As Prawat (1992) explains, a static view of curriculum is one impediment to significant teacher change:

> Instead of viewing students and curriculum interactively . . . teachers tend to regard them as similar factors that somehow must be reconciled. . . . Teachers focus on the packaging and delivery of content, instead of on more substantive issues of knowledge selection and construction. (p. 389)

If teachers are to view students and curriculum dynamically, they need to learn to make classroom-based developments within the curriculum implementation process. After all, curriculum developers may wish to create certain learning experiences for students, but they cannot fully anticipate how particular students will interact with the mathematical activities. Teachers require support not only in coming to recognize the need to adapt curriculum, but also in learning *how* to adapt it.

Experiences with innovative curriculum materials can directly challenge teachers' beliefs about curriculum. The distinctions between reform-oriented and traditional curricula provide immediate opportunities for teachers to explore, and possibly experience, multiple approaches to mathematical subject matter and mathematics pedagogy. Teachers' recognition of the multiplicity of curricular approaches is critical to their movement toward adopting more innovative instructional practices. As teachers identify and weigh the value of specific characteristics of curriculum, they may be pressed to recognize the need to make contextual, classroom-based decisions about instructional design. When teachers' beliefs about curriculum include an inquiry perspective toward *their own* development of pedagogy, we may see a corresponding increase in teachers' ability and inclination to honor and capitalize on students' processes of mathematical sense-making in the classroom. In other words, just as the quality of students' learning hinges upon their ability to make sense of mathematical problems and situations, teachers' development hinges upon educative opportunities to engage in sense-making and problem-solving about mathematics curriculum.

We should take more seriously the powerful role that curriculum materials can play in the learning of teachers throughout their careers. Most teachers rely upon one or two primary textbooks to guide their classroom instruction. If teachers can learn to use their textbooks for their own personal development, then they will be better prepared to learn from and deal productively with the types of materials that will continue to emerge in school settings in the future. Teachers need guidance in learning to make reasoned pedagogical decisions about how to judiciously incorporate the recommendations of curriculum materials into their own instruction. Such learning must extend beyond making choices among particular practices or

activities to the broader development of sensible and useable theories of teaching and learning. Explicit emphasis in professional development activities on the role of curriculum materials in students' learning will support teachers in more effectively using textbooks and other resource materials to teach themselves and their students in the future.

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